



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

NOTE ON THE TRANSFORMATION OF A DETERMINANT INTO ANY OTHER EQUIVALENT DETERMINANT.

BY THOMAS MUIR, M. A., F. R. S. E.

PROFESSOR Van Velzer's interesting note on the above subject in ANALYST, Vol. IX, pp. 116-118, has recalled to my mind a theorem to which I was led in dealing with the "Transformations connecting General Determinants with Continuants". (Trans. Roy. Soc. Edinb., XXX, pp. 5-14.) Taking determinants of the fourth order, the theorem is as follows:—

The first three elements of the last column (say) of the determinant $\begin{vmatrix} a_1 & b_2 \\ c_3 & d_4 \end{vmatrix}$ may be replaced by any three magnitudes whatever, α, β, γ , provided the fourth element be changed into

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 - \alpha \\ b_1 & b_2 & b_3 & b_4 - \beta \\ c_1 & c_2 & c_3 & c_4 - \gamma \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} \div \begin{vmatrix} a_1 & b_2 & c_3 \end{vmatrix}.$$

For, calling the said fourth element x we are to have

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 & \alpha \\ b_1 & b_2 & b_3 & \beta \\ c_1 & c_2 & c_3 & \gamma \\ d_1 & d_2 & d_3 & x \end{vmatrix} \\ = \begin{vmatrix} a_1 & a_2 & a_3 & \alpha \\ b_1 & b_2 & b_3 & \beta \\ c_1 & c_2 & c_3 & \gamma \\ d_1 & d_2 & d_3 & 0 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 & 0 \\ b_1 & b_2 & b_3 & 0 \\ c_1 & c_2 & c_3 & 0 \\ d_1 & d_2 & d_3 & x \end{vmatrix}$$

$$\therefore \begin{vmatrix} a_1 & a_2 & a_3 & a_4 - \alpha \\ b_1 & b_2 & b_3 & b_4 - \beta \\ c_1 & c_2 & c_3 & c_4 - \gamma \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} = x \begin{vmatrix} a_1 & b_2 & c_3 \end{vmatrix}$$

$$\text{and } \therefore x = \begin{vmatrix} a_1 & a_2 & a_3 & a_4 - \alpha \\ b_1 & b_2 & b_3 & b_4 - \beta \\ c_1 & c_2 & c_3 & c_4 - \gamma \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} \div \begin{vmatrix} a_1 & b_2 & c_3 \end{vmatrix}$$

as was to be shown.

The condition for the possibility of effecting the transformation is, as before, indicated by the occurrence of $\begin{vmatrix} a_1 & b_2 & c_3 \end{vmatrix}$ as a *divisor* in the value of x .

Applying the theorem to the case of the transformation of

$$\begin{vmatrix} 0 & b & c \\ b & 1 & a \\ c & a & 1 \end{vmatrix} \quad \text{into} \quad \begin{vmatrix} 2abc & b & c \\ b & 1 & 0 \\ c & 0 & 1 \end{vmatrix}$$

we first change the column $c, a, 1$ into

$$c, 0, \begin{vmatrix} 0 & b & 0 \\ b & 1 & a \\ c & a & 1 \end{vmatrix} \div (-b^2),$$

i. e., into

$$c, 0, 1-(ac \div b);$$

and so on, exactly as Professor Van Velzer does.

This example fortunately is easy, and the process as applied to it appears to the best advantage. It is desirable however to see the shady side as well, and for this purpose I give the curious identity

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ a_5+b_2 & a_1+b_3 & a_2+b_4 & a_3+b_5 & a_4+b_1 \\ a_4+b_3 & a_5+b_4 & a_1+b_5 & a_2+b_1 & a_3+b_2 \\ a_3+b_4 & a_4+b_5 & a_5+b_1 & a_1+b_2 & a_2+b_3 \\ a_2+b_5 & a_3+b_1 & a_4+b_2 & a_5+b_3 & a_1+b_4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ a_5-b_2 & a_1-b_3 & a_2-b_4 & a_3-b_5 & a_4-b_1 \\ a_4-b_3 & a_5-b_4 & a_1-b_5 & a_2-b_1 & a_3-b_2 \\ a_3-b_4 & a_4-b_5 & a_5-b_1 & a_1-b_2 & a_2-b_3 \\ a_2-b_5 & a_3-b_1 & a_4-b_2 & a_5-b_3 & a_1-b_4 \end{vmatrix}$$

which possesses considerable interest in the theory of alternants.

Bishopton, Glasgow, Scotland, Oct. 1882.

INTEGRATION OF SOME GENERAL CLASSES OF TRIGONOMETRIC FUNCTIONS.

BY PROF. P. H. PHILBRICK, IOWA STATE UNIVERSITY, IOWA CITY.

[Continued from page 180, Vol. IX.]

$$\begin{aligned} \therefore \int \frac{dx}{(a+b \sec x)^n} &= \int \frac{a dx}{(a+b \sec x)^{n+1}} - \frac{\tan x \sec x}{(a+b \sec x)^{n+1}} + (n+1)b \\ &\times \int \frac{\sec^2 x dx}{(a+b \sec x)^{n+2}} + 2 \int \frac{\sec^3 x dx}{(a+b \sec x)^{n+1}} - (n+1)b \int \frac{\sec^4 x dx}{(a+b \sec x)^{n+2}}. \end{aligned}$$

Now

$$\begin{aligned} \frac{\sec^2 x}{(a+b \sec x)^{n+2}} &= \frac{1}{b^2} \left[\frac{1}{(a+b \sec x)^n} - \frac{2a}{(a+b \sec x)^{n+1}} + \frac{a^2}{(a+b \sec x)^{n+2}} \right] \\ \frac{\sec^3 x}{(a+b \sec x)^{n+1}} &= \frac{1}{b^3} \left[\frac{1}{(a+b \sec x)^{n-2}} - \frac{3a}{(a+b \sec x)^{n-1}} + \frac{3a^2}{(a+b \sec x)^n} \right. \\ &\quad \left. - \frac{a^3}{(a+b \sec x)^{n+1}} \right] \\ \frac{\sec^4 x}{(a+b \sec x)^{n+2}} &= \frac{1}{b^4} \left[\frac{1}{(a+b \sec x)^{n-2}} - \frac{4a}{(a+b \sec x)^{n-1}} + \frac{6a^2}{(a+b \sec x)^n} \right. \\ &\quad \left. - \frac{4a^3}{(a+b \sec x)^{n+1}} + \frac{a^4}{(a+b \sec x)^{n+2}} \right]. \end{aligned}$$